MaTHink 2012

Mathematical Modeling

March 31, 2012

10:45 a.m. – 12:15 p.m.

Presenter

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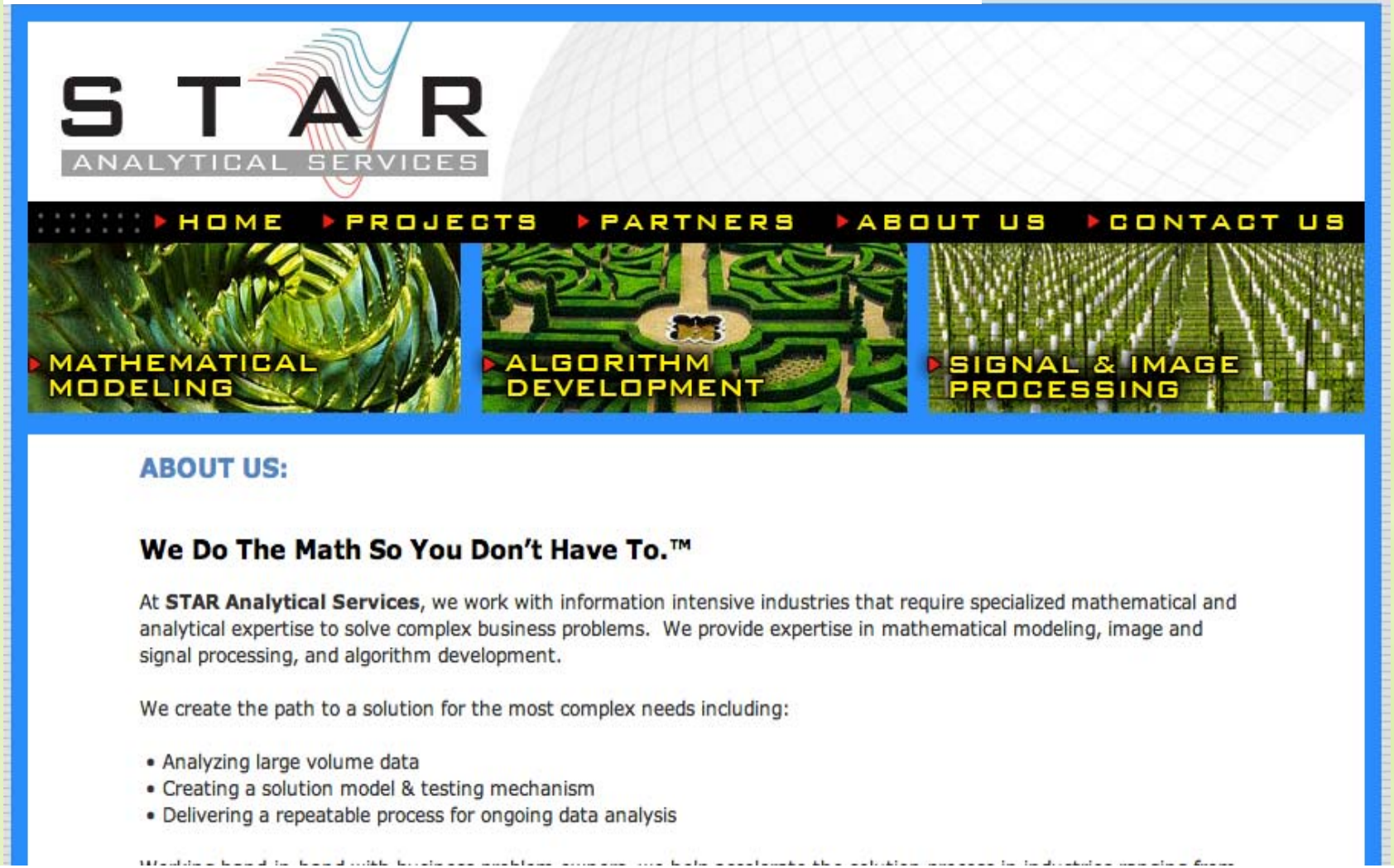
STAR Analytical Services

“We do the math so you don’t have to.”.”

http://www.staranalyticalservices.com/about_us.html



Agenda

- What is mathematical modeling?
- How do we do it?
- Let’s do it!
- Challenges to Modeling
- Resources for Modeling

Key Question

What is mathematical modeling?

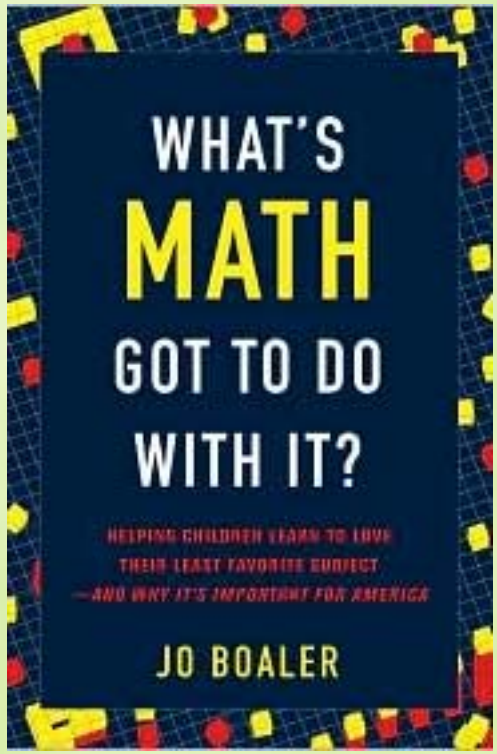
When am I ever…?

What mathematics should students learn in school?

What should students be able to do with the mathematics they learn?

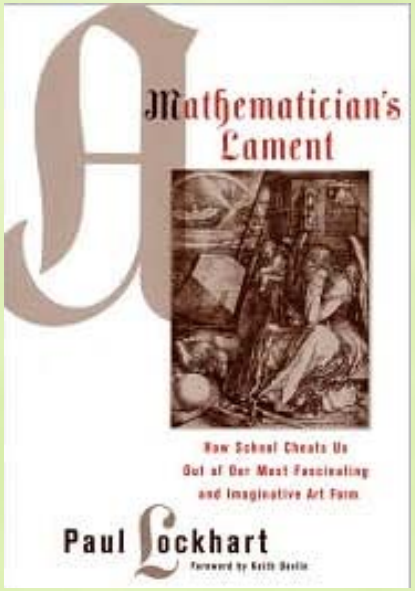
“Whether browsing the Web, interpreting medical records, administering medicine, reading the news, working with finances, or taking part in elections, twenty-first-century citizens need mathematics. But the mathematics that people need is not the sort of math learned in most classrooms. People do not need to regurgitate hundreds of standard methods. They need to reason and problem solve, flexibly applying methods in new situations.”

Jo Boaler, What’s Math Got to Do With It, p.7.



"Mathematics *is the art of explanation*. If you deny students the opportunity to engage in this activity—to pose their own problems, to make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs—you deny them mathematics itself. So no, I'm not complaining about the presence of facts and formulas in our mathematics classes, I'm complaining about the lack of *mathematics* in our mathematics classes."

Paul Lockhardt, Mathematician's Lament, p.29.



Smarter Balanced Assessment Consortium

Draft Content Specifications

Claim #1: CONCEPTS AND PROCEDURES (40%)
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Claim #2: PROBLEM SOLVING (20%)
Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies. (The challenge is in identifying or using an appropriate solution path.)

Claim #3: COMMUNICATING REASONING (20%)
Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Claim #4: MODELING AND DATA ANALYSIS (20%)
Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Rationale for Claim #4

Smarter Balanced Assessment Consortium
Draft Content Specifications

Modeling is the bridge across the “school math”/”real world” divide that has been **missing** from many mathematics curricula and assessments.

In the real world problems do not come neatly “packaged.” Real world problems are complex, and often contain insufficient or superfluous data. Assessment tasks will involve formulating a problem that is tractable using mathematics—that is, formulating a model. This will usually involve making assumptions and simplifications. Students will need to select from data at hand, or estimate data that are missing. (Such tasks are therefore distinct from the problem solving tasks described in Claim #2, that are well-formulated.)

What is mathematical modeling according to the California Common Core State Standards?

Standards for Mathematical Practice, CaCCSS

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

What is mathematical modeling?

"Mathematical modeling is the process by which students encounter a real-world situation, pose a question regarding the situation, and use mathematics to seek an answer to the question. Mathematical modeling involves identifying the relevant features symbolically, analyzing the model, and considering the accuracy and limitations of that model."

NCTM, 2000.

What is mathematical modeling?

According to Joshua Paul Abrams, a teacher of mathematical modeling:

"Mathematical modeling is the process of using mathematics to study a question from outside the field of mathematics. A mathematical model is a representation of a particular phenomenon using structures such as graphs, equations or algorithms."

Joshua Paul Abrams, Mathematical Modeling, 2001. p.2..

"They [students] should spend considerable time using math to resolve problems for which mathematics is neither the initial source of their motivation nor the end result. When we teach mathematics as an intellectual lens through which non-mathematical questions can be examined, then we are providing our students with an education that will serve them and our society throughout their lives."

Joshua Paul Abrams, Mathematical Modeling, 2001, p.4.

STAR Systems

"We do the math so you don't have to."

Why mathematical modeling? Mathematical modeling translates identified issues or problems within a system and breaks it down into usable, mathematical formulations which through mathematical theory and analysis provide another way of looking at the system. The end goal is to gain deeper insight into the system, and to formulate potential ideas and solutions. It generally moves through these phases:

- Precise breakdown of the problem.
- A complete and thorough understanding of the system that is being modeled.
- Creating a pathway for improved design or control.

Mathematical Modeling

2010/2011 Mathematical Modeling Task Force

"Mathematical modeling is the link between mathematics and the rest of the world."

Meerschaert, M., Mathematical Modeling, Elsevier Science, 2010

The process of beginning with a situation and gaining understanding about that situation is generally referred to as "modeling." If the understanding comes about through the use of mathematics, the process is known as mathematical modeling.

Mathematical Modeling

2010/2011 Mathematical Modeling Task Force

Step 1. Identify a situation. Notice something that you wish to understand, and pose a well-defined question indicating exactly what you wish to understand.

Mathematical Modeling

2010/2011 Mathematical Modeling Task Force

Step 2. Simplify the situation. List the key features (and relationships among those features) that you wish to include for consideration. These are the assumptions on which your model will rest. Also note features and relationships you choose to ignore for now.

Mathematical Modeling

2010/2011 Mathematical Modeling Task Force

Step 3. Build the model. Solve the problem. Interpret in mathematical terms the features and relationships you have chosen. (Define variables, write equations, draw shapes, measure objects, calculate probabilities, gather data and organize into tables, make graphs, etc.). That is the model. Then, apply the model and solve the problem. (Solve the equation, draw inferences from patterns in the data, compare results to a standard result, etc.)

Mathematical Modeling

2010/2011 Mathematical Modeling Task Force

Step 4. Evaluate and revise the model. Go back to the original situation and see if results of mathematical work make sense. If so, use the model until new information becomes available or assumptions change. If not, reconsider the assumptions you made in step 2 and revise them to be more realistic.

What is mathematical modeling?

According to Henry Pollak:

"Every application of mathematics uses mathematics to understand or evaluate or predict something in the part of the world outside of mathematics. What distinguishes modeling from other forms of applications of mathematics are…

(1) *explicit* attention at the beginning of the process of getting from the problem outside of mathematics to its mathematical foundation and

(2) an *explicit* reconciliation between the mathematics and the real-world situation at the end.

Throughout the modeling process, consideration is given to both the external world and the mathematics, and the results have to be both mathematically correct and reasonable in a real-world context."

Henry Pollak, A History of the Teaching of Modeling.

Modeling In a Nutshell (draft)

Start With Life

-- Narrow or Broad – (one path, one solution or several paths, many possible solutions)
-- Well-defined question or task vs. open to refinement
-- Life can be complex → Simplify!

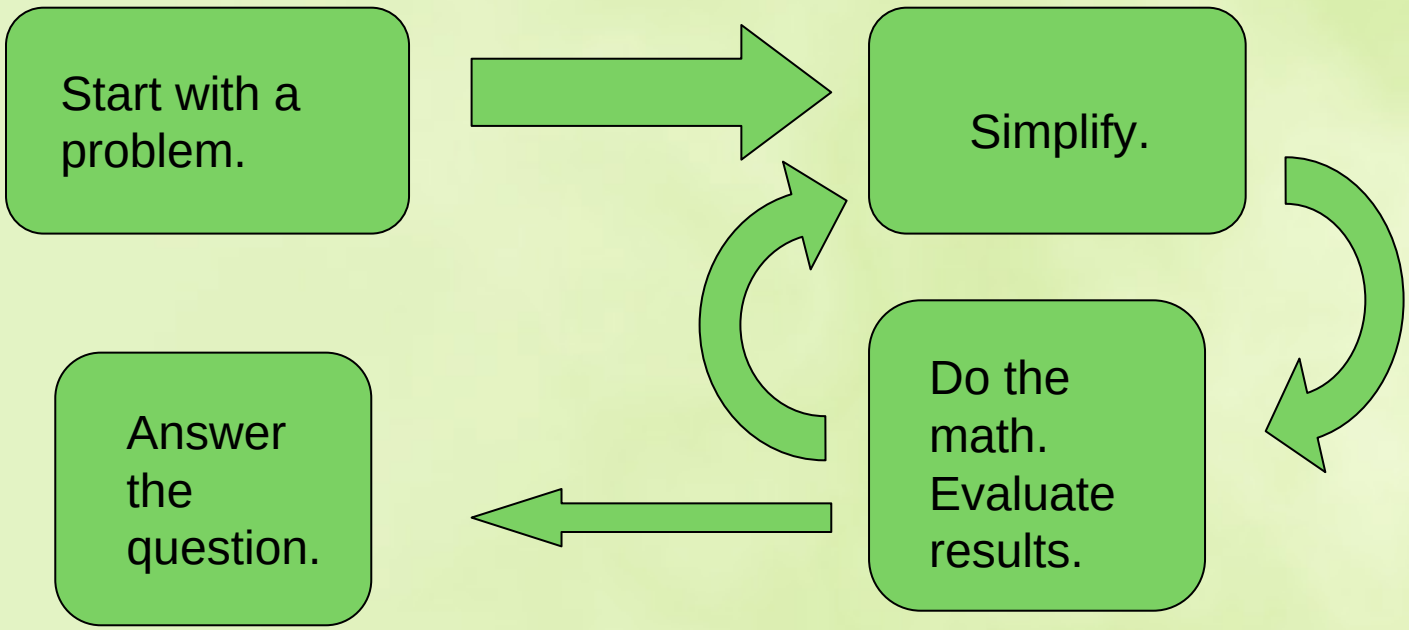
Do The Math!

-- Use available tools, models, representations.
-- Learn new ones (and the requisite mathematics)
-- Attend to accuracy, appropriateness, relevance

Check Back With Life

-- Apply, revise, extend.

Mathematical Modeling Process



Five symptoms we are doing math reasoning wrong…

1. Lack of initiative.
2. Lack of perseverance
3. Lack of retention
4. Aversion to Word Problems
5. Eagerness for Formula

Dan Meyer, TED Talk

Five suggestions to support students…

1. Use multimedia
2. Encourage student intuition
3. Ask the shortest question you can
4. Let students build the problem
5. Be less helpful

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Snow Cones

What question might you ask based on the context of snow cones?

Snow Cones

Should you rent a booth and sell snow cones at Market Night in Redlands?

Snow Cones

How many snow cones would you have to sell at Market Night in Redlands if you wish to make a profit of $200?

Snow Cones

Space Rental = $50.00

Selling Price = $2.50

Ice = $1.99 for 20 lbs.

Cups = $9.00 for 200

Syrup = $6.50 per qt.

Snow Cone = 12 oz. crushed ice + 2 oz. syrup

Modeling and Teaching

Modeler and Modeling

- Begins with life. (Math provides the tools and pathways.)
- Focus on understanding or solving a life problem or improving a product or situation.
- Results in a solution, decision, recommendation, modification, plan

Teacher/Problem Writer

- Begins with mathematics (content standard).
- Focus on mathematics and uses applications to illustrate, motivate, develop or understand mathematics.
- Results in deeper understanding of the mathematics.

Modeling in the Math Classroom

Problems

- Easiest to incorporate in the classroom.
- Should I use the 20% off coupon or the $10 off coupon?

Lessons

- How should I pack the little boxes into the big box?
- SBAC: Taxicab Problem, Long Jump Problem, MARS

Units

- Extended commitment of time.
- What is the most efficient way to package soda cans?

Curriculum/Course

Where do I find real life problems for modeling?

Loss Prevention

A large department store is a target for shoplifters. What can the store do to minimize losses?

Coupon Choice

Karen had two coupons when she bought her shoes at the department store. The clerk said the $10.00 off coupon is usually the best. Was the clerk correct?

Buying a Car

The auto dealer is offering a cash rebate of $1,000 or attractive interest rates of 1.9% for a 36-month loan, 3.9% for a 48-month loan and 5.9% for a 60-month loan. What is the best choice?

Trauma Center

A trauma center serves several communities. Suppose a group of small towns agree to fund and build a regional trauma center. How do you determine the best location?

Math Challenge #44 TABLE FOR 19

How much room do you need at a table?

Figure This! Math Challenges for Families

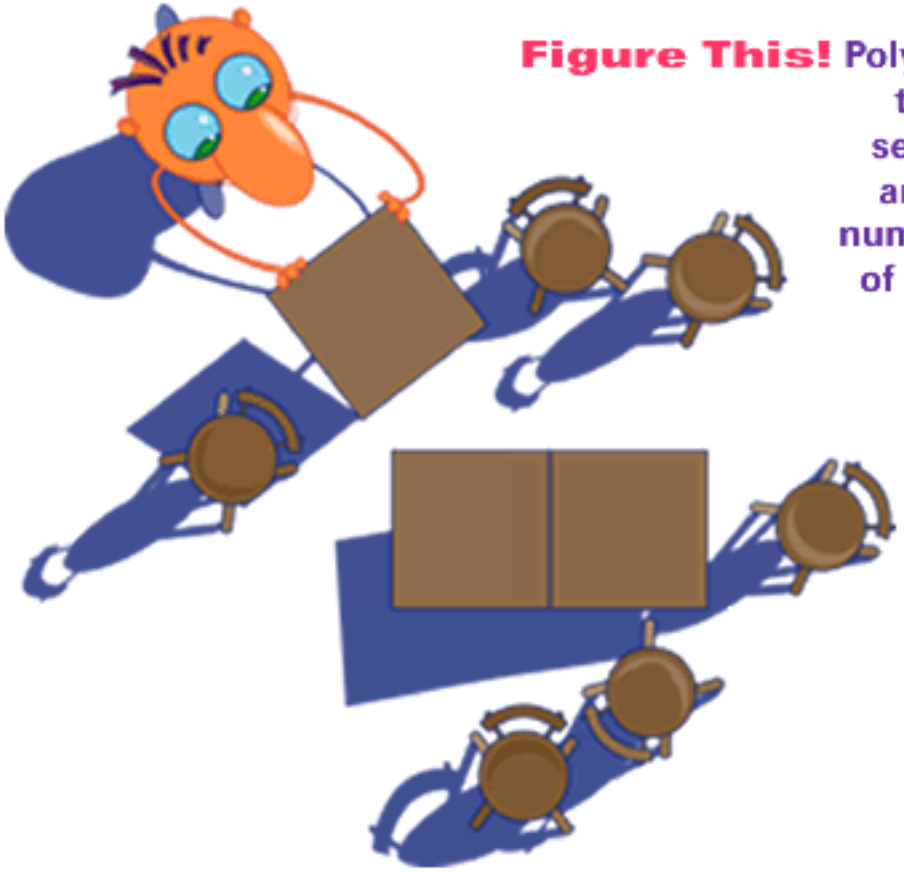

Figure This! Polygon' s Restaurant has square tables that seat one person on each side. To seat larger parties, two or more tables are pushed together. What is the least number of tables needed to seat a party of 19 people who want to sit together?

Hint?

Finding patterns and arranging geometric shapes are used by architects, landscapers, quiltmakers, and carpet layers in their work..

Pizza Deals

What size pizza is the best deal?

Medium Cheese = $9.99
Large Cheese = $11.99
X-Large Cheese = $13.99

Medium = 12” diameter
Large = 14” diameter
X-Large = 16” diameter

	May	Jun	Jul	Aug	Sept	Oct
Brand X	60	45	35	25	20	15
Brand Y	35	30	30	25	25	22

1. Use the safety helmet data to make a line graph for Brand X.
2. Use the line graph from Exercise 1 and the safety-helmet data to make a double line graph.
3. **Analyze** What conclusions can you draw from the graph?

	May	Jun	Jul	Aug	Sept	Oct
Jim's	60	45	35	25	20	15
John's	35	30	30	25	25	22

Which helmet is better?

Colonial Population Growth		
	North	South
1640	26,634	26,037
1670	111,935	107,400
1700	250,888	223,071
1740	905,563	755,539
1770	2,148,076	1,688,254

1. How many more people lived int he North than the South in 1770?
2. What was the total population of the North and the South in 1770?
3. **Challenge** Make a double line graph to show the population growth. Label the x-axis in 50Õs from 1600 to 1800. Label the y-axis in 100,000s from 0 to 2,200,000.

Colonial Population Growth		
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How many people lived in the North and South in 1600 (or 1800)?

According to the data, how many people would be living in the North and South in 2010?

Is the population of our country growing at the same rate as it was in the 1600-1700s?

Thank you for coming!

- This powerpoint will be available at the following website:
- http://algebraforum.wikispaces.com/MaTHink2012

If you would like a copy of additional mathematical modeling resources, please email me at:

Jennifer.R.Hagman “at” gmail.com